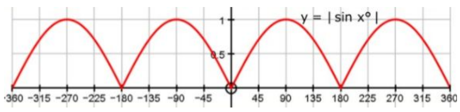
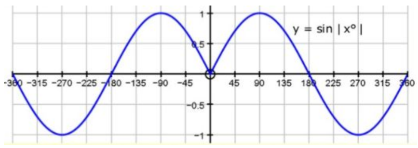
MATHS-PURE

Year 13 Knowledge Organiser Pure

1. Algebraic Methods

A rational number can be written as $\frac{a}{b}$, where…	a and b are integers

2. Functions and Graphs

Graph of $y = \lvert f(x) \rvert$	Everything underneath the x axis is reflected up to join graph above x axis Eg. 
Graph of $y = f\lvert(x)\rvert$	Everything on the right of the y axis stays the same, graph on the left of y axis is replaced by reflection from right side Eg. 
Solving inequalities involving modulus	1. Sketch both sides of the inequality 2. Solve the inequality as an equation 3. Use the graph and solutions of equation to find region which satisfies inequality
When would a function not have an inverse?	'Because it is not a one-to-one function'
$ff^{-1}(x) = f^{-1}f(x) =$	x
How do I sketch $f^{-1}(x)$?	Reflect the graph of f(x) in the line y = x
The domain of f(x) is the range of	$f^{-1}(x)$
The range of f(x) is the domain of	$f^{-1}(x)$
Translation $(a\ 0)$	$f(x) \to f(x-a)$
Translation $(0\ b)$	$f(x) \to f(x) + b$
Stretch scale factor c in the x direction	$f(x) \to f\left(\frac{x}{c}\right)$
Stretch scale factor d in the y direction	$f(x) \to df(x)$
Reflection in the y axis	$f(x) \to f(-x)$
Reflection in the x axis	$f(x) \to -f(x)$
Composite transformations affect x and y	Order does not matter
Composite transformations both affect x	Describe translation first
Composite transformations both affect y	Describe stretch first

3. Sequences & Series

Arithmetic sequence a =	First term
Arithmetic sequence d =	Common difference
Arithmetic nth term $U_n =$	a + (n – 1)d
Sum of first n terms $S_n =$	$\frac{n}{2}$ (2a + (n – 1)d)
Sum of first n terms $S_n =$	$\frac{n}{2}(a + l)$, $l = last\ term$
Geometric sequence a =	First term
Geometric sequence r =	Common ratio
Geometric nth term $U_n =$	ar^{n-1}
Sum of first n terms $S_n =$	$\frac{a(1-r^n)}{1-r}$, use when lrl < 1
Sum of first n terms $S_n =$	$\frac{a(r^n-1)}{r-1}$, use when lrl > 1
A geometric sequence only converges if…	lrl < 1
Sum to infinity $S_\infty =$	$\frac{a}{1-r}$

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Sequence is increasing if	$U_{n+1} > U_n$ *for all* $n \in N$
Sequence is decreasing if	$U_{n+1} < U_n$ *for all* $n \in N$
A sequence is periodic if	The terms repeat in a cycle
The order k for periodic sequence is such that	$U_{n+k} = U_n$ *for all* $n \in N$

4. **Binomial Expansion**

$(1+x)^n =$	$1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1\times 2\times \ldots \times r}x^r + \ldots$
$(1 + x)^n$ is valid when	$\lvert x\rvert < 1$
$(1 + ax)^n =$	$1 + nax + \frac{n(n-1)}{2!}(ax)^2 + \frac{n(n-1)(n-2)}{3!}(ax)^3 + \ldots\ldots$
$(1 + bx)^n$ is valid when	$\lvert bx\rvert < 1$, $\lvert x\rvert < \frac{1}{\lvert b\rvert}$
$(a + x)^n =$	$a^n(1 + \frac{x}{a})^n$
$(a + x)^n$ is valid when	$\lvert \frac{x}{a} \rvert < 1$, $\lvert x \rvert < a$

5. **Radians**

2π *radians* $=$	$360°$
π *radians* $=$	$180°$
$\frac{\pi}{2}$ *radians* $=$	$90°$
$\frac{\pi}{3}$ *radians* $=$	$60°$
$\frac{\pi}{4}$ *radians* $=$	$45°$
$\frac{\pi}{6}$ *radians* $=$	$30°$
1 *radian* $=$	$\frac{180°}{\pi}$
Sin 0=	0
Sin $\frac{\pi}{6}$ =	$\frac{1}{2}$
Sin $\frac{\pi}{4}$ =	$\frac{\sqrt{2}}{2}$
Sin $\frac{\pi}{3}$ =	$\frac{\sqrt{3}}{2}$
Sin $\frac{\pi}{2}$ =	1
Cos 0=	1
Cos $\frac{\pi}{6}$ =	$\frac{\sqrt{3}}{2}$
Cos $\frac{\pi}{4}$ =	$\frac{\sqrt{2}}{2}$
Cos $\frac{\pi}{3}$ =	$\frac{1}{2}$
Cos $\frac{\pi}{2}$ =	0
Tan 0=	0
Tan $\frac{\pi}{6}$ =	$\frac{1}{\sqrt{3}}$
Tan $\frac{\pi}{4}$ =	1
Tan $\frac{\pi}{3}$ =	$\sqrt{3}$
Tan $\frac{\pi}{2}$ =	∞
Arc length l=	$r\theta$
Area of sector A=	$\frac{1}{2}r^2\theta$
Area of segment=	$\frac{1}{2}r^2(\theta - sin\theta)$
When θ small, $sin\theta \approx$	θ
When θ small, $cos\theta \approx$	$1 - \frac{\theta^2}{2}$
When θ small, $tan\theta \approx$	θ

6. **Trigonometric Functions**

$\sec sec\, x =$	$\frac{1}{\cos cos\, x}$
$x =$	$\frac{1}{\sin sin\, x}$
$\cot cot\, x =$	$\frac{1}{tanx} = \frac{\cos cos\, x}{\sin sin\, x}$

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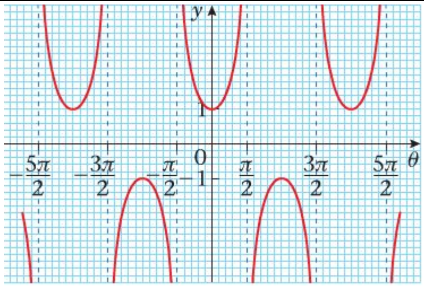
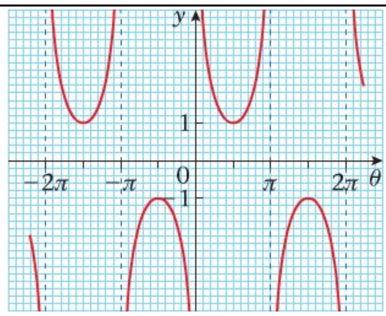
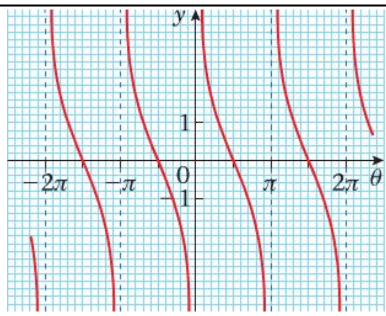
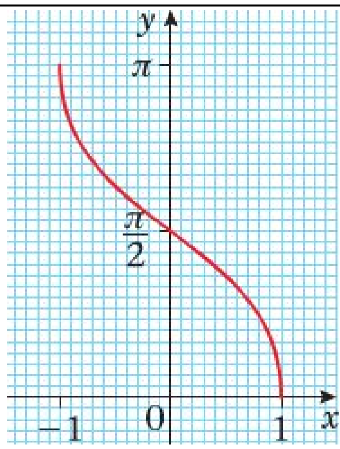
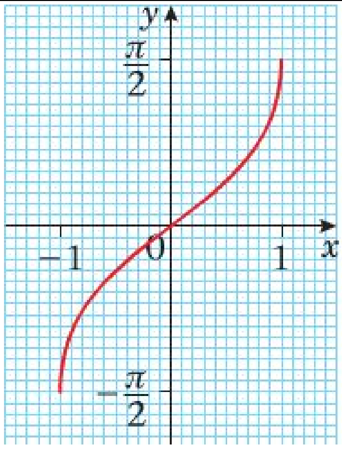
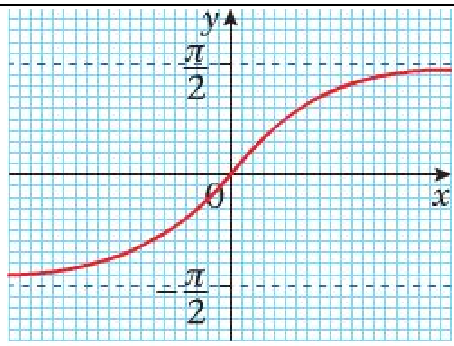
Graph of sec *sec x* =	
Graph ofx =	
Graph of cot *cot x* =	
$1 + tan^2 x =$	$sec^2 x$
$1 + cot^2 x =$	$cosec^2 x$
Graph of arcsin =	

Graph of arccos =



Graph of arctan =



7. **Trigonometry and Modelling**

$\sin(A \pm B) =$	$\sin A \cos B \pm \cos A \sin B$
$\cos(A \pm B) =$	$\cos A \cos B \mp \sin A \sin B$
$\tan(A \pm B) =$	$\dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
sin 2A =	2sinAcosA
cos 2A =	$\cos^2 A - \sin^2 A$
	$2\cos^2 A - 1$
	$1 - 2\sin^2 A$
tan 2A =	$\frac{2 \tan tan A}{1 - tan^2 A}$
$\int \sin x \cos x \, dx =$	$\frac{1}{2}\int \sin 2x$
$\int \cos^2 x \, dx =$	$\frac{1}{2}\int (\cos 2x + 1)\, dx$
$\int sin^2 x \, dx =$	$\frac{1}{2}\int 1 - \cos cos\, 2x \;\; dx$
R=	$\sqrt{A^2 + B^2}$
α =	$tan^{-1}\frac{B}{A}$
(1) $A \sin x + B \cos x \equiv$	$R \sin(x + \alpha)$

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(2) $A\sin x - B\cos x \equiv$	$R\sin(x-\alpha)$
(3) $A\cos x + B\sin x \equiv$	$R\cos(x-\alpha)$
(4) $A\cos x - B\sin x \equiv$	$R\cos(x+\alpha)$
8. **Parametric Equations**	
For y = f(x), x = p(t) and y = q(t) the domain of f(x)=	Range of p(t)
For y = f(x), x = p(t) and y = q(t) the domain of f(x)=	Range of q(t)
9. **Differentiation**	
Chain Rule	Differential of outside function multiplied by differential of inside function
Product Rule If y = uv then $\frac{dy}{dx}=$	$u\frac{dv}{dx}+v\frac{du}{dx}$
Quotient Rule If $y=\frac{u}{v}$ then $\frac{dy}{dx}=$	$\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
f(x)	**f'(x)**
e^{kx}	ke^{kx}
$\ln \ln x$	$\frac{1}{x}$
a^{kx}	$(k\ln a)a^{kx}$
$\sin \sin x$	$\cos \cos x$
$\cos \cos x$	x
$\tan \tan x$	$sec^2 x$
x	$x\ cotx$
$\sec \sec x$	$\sec sec\ xtanx$
$\cot cot\ x$	$-cosec^2 x$
arcsin x	$\frac{1}{\sqrt{1-x^2}}$
arccos x	$-\frac{1}{\sqrt{1-x^2}}$
arctan x	$\frac{1}{1+x^2}$
$\frac{dx}{dy}=$	$\frac{1}{\left(\frac{dy}{dx}\right)}$
Implicit differentiation $\frac{d}{dx}f(y)=$	$f'(x)\frac{dy}{dx}$
Parametric differentiation $\frac{dy}{dx}=$	$\frac{dy}{dt}\div\frac{dx}{dt}$
Function f(x) concave in interval if…	$f''(x)\le 0$
Function f(x) convex in interval if…	$f''(x)\ge 0$
A point of inflection is one where…	$f''(x)$ *changes sign*
10. **Numerical Methods**	
Statement needed to show root α lies between a and b	Sign change, therefore α lies between a and b
Newton-Raphson formula $x_{n+1}=$	$x_n-\frac{f(x_n)}{f'(x_n)}$
11. **Integration**	

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Integration by Parts $\int u \frac{dv}{dx} dx =$	$uv - \int v \frac{du}{dx} dx$
Steps for using Integration by Substitution	1. Find $\frac{du}{dx}$ and rearrange to make dx the subject 2. Replace dx with the above and substitute u according to given substitution 3. Replace any other x terms with expression in u 3. Change limits into terms of u if there are any 4. Integrate with respect to u 5. Evaluate with limits if there are any **OR** put back in terms of x
$\int \frac{f'(x)}{f(x)} dx =$	$ln\|f(x)\| + c$
$f(x)$	$\int f(x)dx$
$\int (ax+b)^n dx =$	$\frac{1}{a(n+1)}(ax+b)^{n+1} + c$
$\int e^{ax+b} dx =$	$\frac{1}{a}e^{ax+b} + c$
$\int \frac{1}{x} dx =$	$\ln ln\ x\ + c$
$\int cosx\ dx =$	$\sin sin\ x\ + c$
$\int sinx\ dx =$	$x\ + c$
$\int sec^2 x\ dx =$	$\tan tan\ x + c$
$\int cosecxcotx\ dx =$	$-\ cosec\ x + c$
$\int secxtanx\ dx =$	$sec\ x + c$
$\int cosec^2 x\ dx =$	$-\ cot\ x + c$
$\int tanx\ dx =$	$ln\ \|\sec sec\ x\ \| + c$
$\int cotx\ dx =$	$ln\ \|sin\| + c$
$\int secx\ dx =$	$ln\ \|\sec sec\ x\ + tanx\| + c$
$\int cosecx\ dx =$	$-\ ln\ \|cosecx + cotx\| + c$
$\int \frac{1}{ax+b}\ dx =$	$\frac{1}{a}\ ln\ \|ax+b\| + c$
$\int \frac{1}{(ax+b)^2}\ dx =$	$-\frac{1}{a(ax+b)}\ + c$
Trapezium rule step length h =	$\frac{b-a}{n}$

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Trapezium rule $\int_a^b y\,dx \approx$	$\frac{h}{2}[y_0 + 2(y_1 + y_2 + \ldots + y_{n-1}) + y_n]$
Separation of variables $When\ \frac{dy}{dx} = g(x)h(y)$	$\int \frac{1}{h(y)} dy = \int g(x)\,dx + A$
12. **Vectors**	
Distance between 2 points $(x_1, y_1, z_1), (x_2, y_2, z_2)$ =	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
Magnitude of vector $(a\ b\ c)$ =	$\sqrt{a^2 + b^2 + c^2}$
Vector **a** makes angle θ_x with positive axis, cos θ_x =	$\frac{x}{\lvert a \rvert}$
Vector **a** makes angle θ_y with positive axis, cos θ_y =	$\frac{y}{\lvert a \rvert}$
Vector **a** makes angle θ_z with positive axis, cos θ_z =	$\frac{z}{\lvert a \rvert}$